Shear-current effect in a turbulent convection with a large-scale shear

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The shear-current effect in a nonrotating homogeneous turbulent convection with a large-scale constant shear is studied. The large-scale velocity shear causes anisotropy of turbulent convection, which produces the mean electromotive force $\boldsymbol{\mathcal{E}}^{(W)} \propto \mathbf{W} \times \mathbf{J}$ and the mean electric current along the original mean magnetic field, where $\mathbf{W}$ is the background mean vorticity due to the shear and $\mathbf{J}$ is the mean electric current. This results in a large-scale dynamo even in a nonrotating and nonhelical homogeneous sheared turbulent convection, whereby the α effect vanishes. It is found that turbulent convection promotes the shear-current dynamo instability, i.e., the heat flux causes positive contribution to the shear-current effect. However, there is no dynamo action due to the shear-current effect for small hydrodynamic and magnetic Reynolds numbers even in a turbulent convection, if the spatial scaling for the turbulent correlation time is $\tau(k) \propto k^{-2}$, where k is the small-scale wave number.

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I. INTRODUCTION

It is generally believed that the large-scale magnetic fields of the Sun, solar-type stars, and galaxies are originated by a dynamo process, i.e., due to a joint action of small-scale helical turbulent motions (the α effect) and large-scale differential rotation (see Refs. [1–9]). It has been recently recognized [10–15] that in a sheared nonhelical and nonrotating homogeneous turbulence whereby the α effect vanishes, the mean-field dynamo is possible due to shear-current effect.

The mechanism of the shear-current dynamo is as follows. The deformations of the original nonuniform magnetic field lines are caused by upward and downward turbulent eddies. In a sheared turbulence the inhomogeneity of the original mean magnetic field breaks a symmetry between the influence of the upward and downward turbulent eddies on the mean magnetic field. This creates the mean electric current $\mathbf{J}$ along the mean magnetic field $\mathbf{B}$ and produces the shear-current dynamo. In particular, the large-scale velocity shear creates anisotropy of turbulence. This produces the mean electromotive force $\boldsymbol{\mathcal{E}}^{(W)} \propto \mathbf{W} \times \mathbf{J}$, where $\mathbf{W}$ is the background mean vorticity due to the shear. Joint effects of the mean electromotive force $\boldsymbol{\mathcal{E}}^{(W)}$ and stretching of the mean magnetic field due to the large-scale shear motions cause the mean-field dynamo instability.

A sheared turbulence is a universal feature in astrophysics. The shear-current effect might be an origin for the large-scale magnetic fields in colliding protogalactic clouds and in merging protostellar clouds [15]. This effect might be also important in accretion discs where the mean velocity shear comes together with rotation, so that both the shear-current effect and the α effect might operate. Since the shear-current effect is not quenched (see Refs. [11,16]) contrary to the quenching of the nonlinear α effect, the shear-current effect might be the only surviving effect, and it can explain the origin of large-scale magnetic fields in astrophysical plasmas with large-scale sheared motions.

The shear-current effect is a fundamental phenomenon which should be studied in different situations, e.g., in different types of turbulence. The goal of the present study is to investigate the shear-current effect in a nonrotating homogeneous turbulent convection with a large-scale constant velocity shear. Note also that in many astrophysical applications turbulent convection plays an important role, e.g., in the convective zones of the Sun and solar-type stars. We have shown that turbulent convection promotes the shear-current dynamo instability. In particular, the heat flux causes positive contribution to the shear-current effect. However, the shear-current dynamo is impossible for small hydrodynamic or magnetic Reynolds numbers even in a turbulent convection, if the spatial scaling for the turbulent correlation time is $\tau(k) \propto k^{-2}$, where k is the small-scale wave number.

This paper is organized as follows. In Sec. II we formulate the governing equations, the assumptions and the procedure of the derivations. In Sec. III we study properties of the shear-current effect in a sheared turbulent convection and discuss the shear-current dynamo. In Sec. IV we draw concluding remarks and perform comparison of the theoretical predictions with the direct numerical simulations. Finally, in the Appendix we perform a detailed derivation of the shear-current effect in a turbulent convection.

II. THE GOVERNING EQUATIONS

In order to study the shear-current effect in a turbulent convection we use a procedure which is similar to that applied in [11,17]. In particular, we employ a mean-field approach whereby the pressure, entropy, velocity, and magnetic fields are separated into the mean and fluctuating parts, where the fluctuating parts have zero mean values. To determine the effect of shear on a turbulent convection we use equations for fluctuations of velocity, magnetic field, and entropy

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$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{U}\cdot\boldsymbol{\nabla})\mathbf{u} - (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{U} - \boldsymbol{\nabla}\left(\frac{p}{\rho_0}\right) - \mathbf{g}s + \frac{1}{\rho_0}[(\mathbf{b}\cdot\boldsymbol{\nabla})\mathbf{B} + (\mathbf{B}\cdot\boldsymbol{\nabla})\mathbf{b}] + \mathbf{u}^N, \tag{1}$$

$$\frac{\partial \mathbf{b}}{\partial t} = (\mathbf{B}\cdot\boldsymbol{\nabla})\mathbf{u} - (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{B} + (\mathbf{b}\cdot\boldsymbol{\nabla})\mathbf{U} - (\mathbf{U}\cdot\boldsymbol{\nabla})\mathbf{b} + \mathbf{b}^N, \tag{2}$$

$$\frac{\partial s}{\partial t} = -\frac{N^2}{g}(\mathbf{u}\cdot\mathbf{e}) - (\mathbf{U}\cdot\boldsymbol{\nabla})s + s^N, \tag{3}$$

where $\mathbf{B}$, $\mathbf{U}$, and S are the mean magnetic field, the mean velocity field, and the mean entropy, $\mathbf{u}$, $\mathbf{b}$, and s are fluctuations of velocity, magnetic field, and entropy, ρ_0 is the fluid density, $N^2 = -\mathbf{g}\cdot\boldsymbol{\nabla}S$, and $\mathbf{g}$ is the acceleration of gravity, $\mathbf{e}$ is the unit vector directed opposite to $\mathbf{g}$, the magnetic permeability of the fluid is included in the definition of the magnetic field, p are the fluctuations of total (hydrodynamic and magnetic) pressure, $\mathbf{v}^N$, $\mathbf{b}^N$, and s^N are the nonlinear terms which include the molecular viscous and diffusion terms. Equations (1)–(3) for fluctuations of fluid velocity, entropy, and magnetic field are written in the Boussinesq approximation. We consider the hydrostatic nearly isentropic basic reference state. The turbulent convection is regarded as a small deviation from a well-mixed adiabatic reference state.

Using Eqs. (1)–(3) written in a Fourier space we derive equations for the instantaneous two-point second-order correlation functions of the velocity fluctuations $\langle u_i u_j\rangle$, the magnetic fluctuations $\langle b_i b_j\rangle$, the entropy fluctuations $\langle ss\rangle$, the cross-helicity tensor $\langle b_i u_j\rangle$, the turbulent heat flux $\langle s u_i\rangle$ and $\langle s b_i\rangle$. The equations for these correlation functions are given by Eqs. (A2)–(A7) in the Appendix. We split the tensor of magnetic fluctuations into nonhelical, $h_{ij} = \langle b_i b_j\rangle$, and helical, $h_{ij}^{(H)}$, parts. The helical part $h_{ij}^{(H)}$ depends on the magnetic helicity, and it is determined by the dynamic equation which follows from the magnetic helicity conservation arguments (see, e.g., Refs. [18–25], and a review [9]).

The second-moment equations include the first-order spatial differential operators $\hat{\mathcal{N}}$ applied to the third-order moments $M^{(\mathrm{III})}$. A problem arises how to close the system, i.e., how to express the set of the third-order terms $\hat{\mathcal{N}}M^{(\mathrm{III})}$ through the lower moments $M^{(\mathrm{II})}$ (see, e.g., Refs. [26–28]). We use the spectral τ approximation which postulates that the deviations of the third-moment terms, $\hat{\mathcal{N}}M^{(\mathrm{III})}(\mathbf{k})$, from the contributions to these terms afforded by the background turbulent convection, $\hat{\mathcal{N}}M^{(\mathrm{III},0)}(\mathbf{k})$, are expressed through the similar deviations of the second moments, $M^{(\mathrm{II})}(\mathbf{k}) - M^{(\mathrm{II},0)}(\mathbf{k})$,

$$\hat{\mathcal{N}}M^{\mathrm{III}}(\mathbf{k}) - \hat{\mathcal{N}}M^{(\mathrm{III},0)}(\mathbf{k}) = -\frac{1}{\tau(k)}[M^{(\mathrm{II})}(\mathbf{k}) - M^{(\mathrm{II},0)}(\mathbf{k})], \tag{4}$$

(see Refs. [11,28–31]), where $\tau(k)$ is the scale-dependent relaxation time, which can be identified with the correlation time of the turbulent velocity field. The quantities with the superscript (0) correspond to the background shear-free turbulent convection with a zeromean magnetic field. We apply the spectral τ approximation only for the nonhelical part h_{ij} of the tensor of magnetic fluctuations. Note that a justification of the τ approximation for different situations has been performed in numerical simulations and analytical studies in [9,11,32–36].

We assume that the characteristic time of variation of the mean magnetic field $\mathbf{B}$ is substantially larger than the correlation time $\tau(k)$ for all turbulence scales. This allows us to get a stationary solution for the equations for the second-order moments, $M^{(\mathrm{II})}$. We split all second-order correlation functions, $M^{(\mathrm{II})}$, into symmetric $h_{ij}^{(s)} = [h_{ij}(\mathbf{k}) + h_{ij}(-\mathbf{k})]/2$ and antisymmetric $h_{ij}^{(a)} = [h_{ij}(\mathbf{k}) - h_{ij}(-\mathbf{k})]/2$ parts with respect to the wave vector $\mathbf{k}$. For the integration in $\mathbf{k}$-space we must specify a model for the background shear-free turbulent convection (i.e., a turbulent convection with $\mathbf{B}=0$). The background turbulent convection is maintained by an imposed vertical heat flux $F_z^* = \langle s u_z\rangle$ with div $\mathbf{F}^* = 0$ at a low boundary of convective region. We used the following model for the homogeneous background turbulent convection:

$$\langle u_i u_j\rangle^{(0)}(\mathbf{k}) = \langle \mathbf{u}^2\rangle P_{ij}(k) W(k), \tag{5}$$

$$\langle b_i b_j\rangle^{(0)}(\mathbf{k}) = \langle \mathbf{b}^2\rangle P_{ij}(k) W(k), \tag{6}$$

$$\langle s u_i\rangle_i^{(0)}(\mathbf{k}) = 3\langle s u_z\rangle e_m P_{im}(k) W(k), \tag{7}$$

where $P_{ij}(k) = \delta_{ij} - k_i k_j/k^2$, δ_{ij} is the Kronecker tensor, $W(k) = E(k)/8\pi k^2$, the energy spectrum is $E(k) = k_0^{-1}(q-1) \times (k/k_0)^{-q}$, $k_0 = 1/l_0$ and the length l_0 is the maximum scale of turbulent motions. The turbulent correlation time is $\tau(k) = C\tau_0(k/k_0)^{-\mu}$, where the coefficient $C = (q-1+\mu)/(q-1)$. This value of the coefficient C corresponds to the standard form of the turbulent diffusion coefficient in the isotropic case, i.e., $\int \tau(k)\langle u_i u_j\rangle^{(0)}(\mathbf{k}) d\mathbf{k} = \eta_T \sigma_{ij}$, where $\eta_T = \tau_0\langle \mathbf{u}^2\rangle/3$. Here the time $\tau_0 = l_0/\sqrt{\langle \mathbf{u}^2\rangle}$ and $\sqrt{\langle \mathbf{u}^2\rangle}$ is the characteristic turbulent velocity in the scale l_0. For the Kolmogorov's type background turbulence (i.e., for a turbulence with a constant energy flux over the spectrum), the energy spectrum $E(k) \propto -d\tau/dk$, the exponent $\mu = q-1$ and the coefficient $C=2$. In the case of a turbulence with a scale-independent correlation time, the exponent $\mu=0$ and the coefficient $C=1$. Motions in the background turbulent convection are assumed to be nonhelical. Using the solution of the derived second-moment equations, we determine the contributions to the mean electromotive force, $\mathcal{E}_i^\sigma = \varepsilon_{imn}\int \langle b_n u_m\rangle_{\mathbf{k}} d\mathbf{k}$, caused by the sheared turbulence (see the Appendix), where ε_{ijk} is the fully antisymmetric Levi-Civita tensor. This procedure allows us to determine the contributions to the shear-current effect caused by the sheared turbulent convection.

III. THE SHEAR-CURRENT DYNAMO

We consider a homogeneous turbulent convection with a constant mean velocity shear, $\mathbf{U} = (0, Sx, 0)$ and $\mathbf{W} = (0, 0, S)$. We consider a most simple form of the mean magnetic field,

$\mathbf{B}=(B_x(z),B_y(z),0)$. The contributions to the mean electromotive force caused by the sheared turbulence, are $\mathcal{E}_i^\sigma=b_{ijk}^\sigma\nabla_k B_j$, where the tensor $b_{ijk}^\sigma=b_{ijk}^u+b_{ijk}^F$ is given by

$$b_{ijk}^u = l_0^2\frac{I_2}{30}\varepsilon_{ikn}[Q_0\nabla_n U_j + 2Q_1(\partial U)_{nj}], \quad (8)$$

$$b_{ijk}^F = a_* l_0^2\frac{I_3}{140}\{Q_3\varepsilon_{ikm}e_{mn}\nabla_n U_j + [Q_2\varepsilon_{ikn} + Q_4(\varepsilon_{ikm}e_{mn}+\varepsilon_{inm}e_{mk})](\partial U)_{nj}\}. \quad (9)$$

Equations (8) and (9) are derived in the Appendix. Here $(\partial U)_{ij}=(\nabla_i U_j+\nabla_j U_i)/2$ and the coefficients Q_n are $Q_0=(3-2\mu)-\epsilon(5+2\mu)$, $Q_1=\epsilon(7+6\mu)-1$, $Q_2=\mu+2$, $Q_3=18-19\mu$, $Q_4=\mu-6$, the parameter $\epsilon=E_m/E_v$, E_m and E_v are the magnetic and kinetic energies per unit mass in the background turbulent convection, and

$$I_2 = \int \tau^2(k)E(k)dk = \frac{(q-1+\mu)^2}{(q-1+2\mu)(q-1)},$$

$$I_3 = \int \tau^3(k)E(k)dk = \frac{(q-1+\mu)^3}{(q-1+3\mu)(q-1)^2}.$$

For the Kolmogorov's type turbulence, the exponent $\mu=q-1$ and the parameters $I_2=4/3$ and $I_3=2$. In the case of a turbulence with a scale-independent correlation time, the exponent $\mu=0$ and the parameters $I_2=I_3=1$. The tensor b_{ijk}^F in Eq. (9) describes the contributions of the heat flux to the shear-current effect, while tensor b_{ijk}^u determines the nonconvective contributions (which are independent of the heat flux) to the shear-current effect. In Eqs. (8) and (9) we have taken into account only the terms which contribute to the shear-current effect. In particular, we have taken into account that $B_y \gg B_x$ [see Eq. (20) below] and considered a weak mean velocity shear $\mathbf{U}=(0,Sx,0)$, where $S\tau_0 \ll 1$. The convective contribution to the dynamo instability due to the shear-current effect depends on the parameter $a_*=2g\tau_0 F_z^*/\langle \mathbf{u}^2\rangle$ which is determined by the budget equation for the total energy. The parameter a_* is given by

$$a_*^{-1} = 1 + \frac{\nu_T(\nabla U)^2 + \eta_T(\nabla B)^2/\rho_0}{g\langle s u_z\rangle}, \quad (10)$$

where ν_T is the turbulent viscosity and η_T is the coefficient of turbulent magnetic diffusion.

Therefore, in the kinematic approximation the mean magnetic field is determined by

$$\frac{\partial B_x}{\partial t} = -\sigma_B S l_0^2 B_y'' + \eta_T B_x'', \quad (11)$$

$$\frac{\partial B_y}{\partial t} = S B_x + \eta_T B_y'', \quad (12)$$

where $B_i''=\partial^2 B_i/\partial z^2$. Here we neglect small contributions to the coefficient of turbulent magnetic diffusion caused by the shear motions because $S\tau_0 \ll 1$. The dimensionless parameter σ_B describes the shear-current effect. Straightforward calculations using Eqs. (8) and (9) yield the parameter $\sigma_B=\sigma_B^u+\sigma_B^F$, where

$$\sigma_B^u = \frac{I_2}{30}(Q_0+Q_1), \quad (13)$$

$$\sigma_B^F = a_*\frac{I_3}{280}(Q_2 - Q_4 + 2(Q_3+Q_4)\sin^2\phi), \quad (14)$$

ϕ is the angle between the unit vector $\mathbf{e}$ and the background vorticity $\mathbf{W}$ due to the large-scale shear. Equations (13) and (14) yield the following final expressions for the parameter σ_B:

$$\sigma_B = \frac{I_2}{15}\left(1 - \mu + \epsilon(1+2\mu) + \frac{a_* 3I_3}{7I_2}[2+3(2-3\mu)\sin^2\phi]\right), \quad (15)$$

where the terms $\propto a_*$ in Eq. (15) describe the contribution of the turbulent convection to the shear-current effect. Equations (11) and (12) determine the shear-current dynamo instability. In particular, the first term $\propto SB_x$ on the right-hand side of Eq. (12) determines the stretching of the magnetic field B_x by the shear motions and produces the field B_y. On the other hand, the interaction of the nonuniform magnetic field B_y with the background vorticity $\mathbf{W}$ produces the electric current along the field B_y. This effect is determined by the first term on the right-hand side of Eq. (11) and causes the generation of the magnetic field component B_x. The growth rate of the mean magnetic field due to the shear-current dynamo instability is given by

$$\gamma = S l_0\sqrt{\sigma_B}K_z - \eta_T K_z^2, \quad (16)$$

where K_z is the large-scale wave number. The necessary condition for the dynamo instability is $\sigma_B > 0$.

The shear-current dynamo instability depends on the spatial scaling of the correlation time $\tau(k)\propto k^{-\mu}$ of the turbulent velocity field, where k is the small-scale wave number. In the absence of turbulent convection, the terms $\propto a_*$ in Eq. (15) vanish, and the shear-current dynamo in a nonconvective turbulence with $\epsilon=0$ occurs for $\mu<1$. For the Kolmogorov's type turbulence, the exponent $\mu=2/3$ and Eq. (15) reads

$$\sigma_B = \tfrac{4}{135}(1 + 7\epsilon + \tfrac{6}{7}a_*). \quad (17)$$

In this case the parameter σ_B is independent of the angle ϕ between the unit vector $\mathbf{e}$ and the background vorticity $\mathbf{W}$. For a turbulent convection with a scale-independent correlation time, the exponent $\mu=0$ and the parameter σ_B is given by

$$\sigma_B = \tfrac{1}{15}[1 + \epsilon + \tfrac{9}{7}a_*(1+3\sin^2\phi)]. \quad (18)$$

In these cases the shear-current dynamo instability causes the generation of the large-scale magnetic field. It follows from Eqs. (15)–(18) that turbulent convection promotes the shear-current dynamo instability. In particular, the heat flux causes positive contribution to the shear-current effect when $2+3(2-3\mu)\sin^2\phi>0$.

However, for small hydrodynamic and magnetic Reynolds numbers, the turbulent correlation time is of the order of

$\tau(k) \propto 1/(\nu k^2)$ or $\tau(k) \propto 1/(\eta k^2)$ depending on the magnetic Prandtl number, i.e., $\tau(k) \propto k^{-2}$. In this case $\mu=2$, and the parameter $\sigma_B<0$ even in a turbulent convection with $\epsilon=0$. This implies that for small hydrodynamic and magnetic Reynolds numbers there is no dynamo action due to the shear-current effect. This result is in agreement with the recent studies [37,38] performed in the framework of the second order correlation approximation (SOCA) for sheared nonconvective flows. This approximation is valid only for small hydrodynamic Reynolds numbers. Even in a high conductivity limit (large magnetic Reynolds numbers), SOCA can be valid only for small Strouhal numbers, while for large hydrodynamic Reynolds numbers (for a developed turbulence), the Strouhal number is 1.

In order to determine the threshold required for the excitation of the shear-current dynamo instability, we consider the solution of Eqs. (11) and (12) with the following boundary conditions: $\mathbf{B}(t,|z|=L)=0$ for a layer of the thickness $2L$ in the z direction. The solution for the mean magnetic field is determined by

$$B_y(t,z) = B_0 \exp(\gamma t)\cos(K_z z + \varphi), \tag{19}$$

$$B_x(t,z) = l_0 K_z \sqrt{\sigma_B} B_y(t,z). \tag{20}$$

For the symmetric mode the angle $\varphi=\pi n$ and the large-scale wave number $K_z=(\pi/2)(2m+1)L^{-1}$, where $n,m=0,1,2,\ldots$. For this mode the mean magnetic field is symmetric relative to the middle plane $z=0$. Let us introduce the dynamo number $D=(l_0 S_*/L)^2\sigma_B$, where parameter $S_*=SL^2/\eta_T$ is the dimensionless shear number. For the symmetric mode the mean magnetic field is generated due to the shear-current effect when the dynamo number $D>D_{\rm cr}=(\pi^2/4)(2m+1)^2$. For the antisymmetric mode the angle $\varphi=(\pi/2)(2n+1)$ with $n=0,1,2,\ldots$, the wave number $K_z=\pi m L^{-1}$ and the magnetic field is generated when the dynamo number $D>D_{\rm cr}=\pi^2 m^2$, where $m=1,2,3,\ldots$. The maximum growth rate of the mean magnetic field in the shear-current dynamo instability, $\gamma_{\max}=S^2 l_0^2 \sigma_B/4\eta_T$, is attained at $K_z=S l_0\sqrt{\sigma_B}/2\eta_T$. Therefore, the characteristic scale of the mean magnetic field variations $L_B=2\pi/K_z=4u_0/(S\sqrt{\sigma_B})$. For the shear-current dynamo, the ratio of the field components B_x/B_y is small [see Eq. (20)]. Remarkably, in the $\alpha\Omega$ dynamo, the poloidal component of the mean magnetic field is much smaller than the toroidal field.

IV. DISCUSSION

In the present study we investigate the shear-current effect in a nonrotating homogeneous turbulent convection with a large-scale constant velocity shear. We show that the condition for the shear-current dynamo is independent of the exponent of the energy spectrum of turbulent convection, but it depends on the scaling exponent μ of the turbulent correlation time $\tau(k) \propto k^{-\mu}$, where k is the small-scale wave number. We discuss three cases in detail: (i) the Kolmogorov's type turbulence with the exponent $\mu=2/3$; (ii) a turbulent convection with a scale-independent correlation time ($\mu=0$); (iii) a turbulent convection with small hydrodynamic and magnetic Reynolds numbers with the scaling $\tau(k) \propto k^{-2}$. We have found that turbulent convection promotes the shear-current dynamo instability. In particular, the heat flux causes positive contribution to the shear-current instability. However, the shear-current dynamo does not occur for small hydrodynamic and magnetic Reynolds numbers even in a turbulent convection, if the spatial scaling for the turbulent correlation time is $\tau(k) \propto k^{-2}$.

For simplicity we consider weak linear velocity shear, $\mathbf{U}=(0,Sx,0)$, where the parameter $S\tau_0 \ll 1$. The main effect of the weak linear velocity shear on turbulent convection is a generation of additional anisotropic velocity fluctuations. We consider turbulent convection in the region which is far from the boundaries, because the constant linear velocity shear cannot exist near the boundaries whereby the boundary layers form. The generation of the magnetic field in a nonlinear velocity shear depends on boundary conditions and requires numerical study. Turbulent convection can be inhomogeneous in this case.

The main goal of this paper is to study an effect of the heat flux on the shear-current dynamo instability in a most simple model of turbulent convection with a linear shear. The shear-current dynamo acts also in inhomogeneous turbulent convection. However, in inhomogeneous turbulence with a large-scale constant velocity shear the kinetic helicity and the α effect do not vanish (see Refs. [10,11,37]). In this case the shear-current dynamo acts together with the α-shear dynamo which is similar to the $\alpha\Omega$ dynamo. The joint action of the shear-current and the α-shear dynamos have been recently discussed in [16,39,40]. The shear-current effect does not quench (see Refs. [11,16]) contrary to the quenching of the nonlinear α effect, the turbulent magnetic diffusion, the effective drift velocity, etc. Therefore, the shear-current effect might be the only surviving effect, and it can explain the origin of large-scale magnetic fields in sheared astrophysical turbulence.

The shear-current dynamo instability is saturated by the nonlinear effects. The nonlinear mean-field dynamo due to a shear-current effect in a nonhelical homogeneous turbulence with a mean velocity shear has been investigated recently in [14] (see also Ref. [40]), whereby the transport of magnetic helicity as a dynamical nonlinearity has been taken into account. The magnetic helicity flux strongly affects the saturated level of the mean magnetic field in the nonlinear stage of the dynamo action. In particular, numerical solutions [14] of the nonlinear mean-field dynamo equations which take into account the shear-current effect, show that if the divergence of the magnetic helicity flux is not small, the saturated level of the mean magnetic field is of the order of the equipartition field determined by the turbulent kinetic energy. These results are in a good agreement with direct numerical simulations [12,13], whereby the generation of the large-scale magnetic field in a nonhelical turbulence with an imposed mean velocity shear has been investigated.

In the direct numerical simulations [12,13] the nonconvective turbulence is driven by a forcing that consists of eigenfunctions of the curl operator with the wave numbers $4.5<k_f<5.5$ and of large-scale component with wave number $k_1=1$. The forcing produces the mean flow $U=U_0\cos(k_1 x)\cos(k_1 x)$. The numerical resolution in these simulations is $128\times512\times128$ mesh points, and the parameters used in these simulations are as following: the magnetic Reynolds number $\mathrm{Rm}=u_{\rm rms}/(\eta k_f)=80$, the magnetic Prandtl number $\mathrm{Pr}_m=\nu/\eta=1$ and $U_0/u_{\rm rms}=5$. The growth rate of the mean magnetic field is about $\gamma\tau_0\approx 2\times10^{-2}$. This allows us to estimate the parameter σ_B characterizing the shear-current effect, $\sigma_B\approx 3.3\times10^{-2}$. On the other hand, our theory predicts $\sigma_B=(3-6)\times10^{-2}$ depending on the parameter μ. Note that in the numerical simulations [12,13] the shear is not small (i.e., the parameter $S\tau_0\sim1$), which explains some difference between the theoretical predictions and numerical simulations. Therefore, the numerical simulations [12,13] clearly demonstrate the existence of the large-scale dynamo in the absence of mean kinetic helicity and alpha effect, in agreement with the theoretical predictions discussed in the present paper.

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APPENDIX: THE ELECTROMOTIVE FORCE IN A SHEARED TURBULENT CONVECTION

In order to study the shear-current effect in a sheared turbulent convection we use a procedure applied in [11,17] for similar problems. Let us derive equations for the second moments. To exclude the pressure term from the equation of motion (1) we calculate $\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{u})$. Then we rewrite the obtained equation and Eqs. (2) and (3) in a Fourier space. We also apply the two-scale approach, e.g., we use large scale $\mathbf{R}=(\mathbf{x}+\mathbf{y})/2$, $\mathbf{K}=\mathbf{k}_1+\mathbf{k}_2$ and small scale $\mathbf{r}=\mathbf{x}-\mathbf{y}$, $\mathbf{k}=(\mathbf{k}_1-\mathbf{k}_2)/2$ variables (see, e.g., Ref. [41]). This implies that we assume that there exists a separation of scales, i.e., the maximum scale of turbulent motions l_0 is much smaller than the characteristic scale L_B of inhomogeneities of the mean magnetic field. We derive equations for the following correlation functions:

$$f_{ij}(\mathbf{k})=\hat{L}(u_i;u_j),\quad h_{ij}(\mathbf{k})=\hat{L}(b_i;b_j),$$

$$g_{ij}(\mathbf{k})=\hat{L}(b_i;u_j),\quad F_i(\mathbf{k})=\hat{L}(s;u_i),$$

$$G_i(\mathbf{k})=\hat{L}(s;b_i),\quad \Theta(\mathbf{k})=\hat{L}(s;s), \tag{A1}$$

where

$$\hat{L}(a;c)=\int\langle a(\mathbf{k}+\mathbf{K}/2)c(-\mathbf{k}+\mathbf{K}/2)\rangle\exp(i\mathbf{K}\cdot\mathbf{R})d\mathbf{K}.$$

The equations for these correlation functions are given by

$$\frac{\partial f_{ij}(\mathbf{k})}{\partial t}=i(\mathbf{k}\cdot\mathbf{B})\Phi_{ij}+I^f_{ij}+I^\sigma_{ijmn}(\mathbf{U})f_{mn}+\hat{\mathcal{N}}f_{ij}, \tag{A2}$$

$$\frac{\partial h_{ij}(\mathbf{k})}{\partial t}=-i(\mathbf{k}\cdot\mathbf{B})\Phi_{ij}+I^h_{ij}+E^\sigma_{ijmn}(\mathbf{U})h_{mn}+\hat{\mathcal{N}}h_{ij}, \tag{A3}$$

$$\frac{\partial g_{ij}(\mathbf{k})}{\partial t}=i(\mathbf{k}\cdot\mathbf{B})[f_{ij}(\mathbf{k})-h_{ij}(\mathbf{k})-h^{(H)}_{ij}]+I^g_{ij}+J^\sigma_{ijmn}(\mathbf{U})g_{mn}+ge_nP_{jn}(k)G_i(-\mathbf{k})+\hat{\mathcal{N}}g_{ij}, \tag{A4}$$

$$\frac{\partial F_i(\mathbf{k})}{\partial t}=-i(\mathbf{k}\cdot\mathbf{B})G_i(\mathbf{k})+I^F_i+H^\sigma_{im}(\mathbf{U})F_m+ge_nP_{\rm in}(k)\Theta(\mathbf{k})+\hat{\mathcal{N}}F_i, \tag{A5}$$

$$\frac{\partial G_i(\mathbf{k})}{\partial t}=-i(\mathbf{k}\cdot\mathbf{B})F_i(\mathbf{k})+I^G_i+(\nabla_mU_i)G_m(\mathbf{k})+\hat{\mathcal{N}}G_i, \tag{A6}$$

$$\frac{\partial\Theta(\mathbf{k})}{\partial t}=-\frac{N^2}{g}F_z(\mathbf{k})+\hat{\mathcal{N}}\Theta, \tag{A7}$$

where hereafter we omit argument t and $\mathbf{R}$ in the correlation functions and neglect small terms $\sim O(\nabla^2)$. Here $\boldsymbol{\nabla}=\partial/\partial\mathbf{R}$, and we also neglect a small term $\propto N^2/g$ in Eq. (A7). In Eqs. (A2)–(A7), $\Phi_{ij}(\mathbf{k})=g_{ij}(\mathbf{k})-g_{ji}(-\mathbf{k})$, $P_{ij}(k)=\delta_{ij}-k_ik_j/k^2$, $\hat{\mathcal{N}}f_{ij}=ge_n[P_{in}(k)F_j(\mathbf{k})+P_{jn}(k)F_i(-\mathbf{k})]+\hat{\mathcal{N}}\tilde{f}_{ij}$, and $\hat{\mathcal{N}}\tilde{f}_{ij}$, $\hat{\mathcal{N}}h_{ij}$, $\hat{\mathcal{N}}g_{ij}$, $\hat{\mathcal{N}}F_i$, $\hat{\mathcal{N}}G_i$, and $\hat{\mathcal{N}}\Theta$ are the third-order moment terms appearing due to the nonlinear terms. The terms which are proportional to the heat flux F_i in the tensor $\hat{\mathcal{N}}f_{ij}$, can be considered as a stirring force for the turbulent convection. Note that a stirring force in the Navier-Stokes turbulence is an external parameter. The tensors $I^\sigma_{ijmn}(\mathbf{U})$, $E^\sigma_{ijmn}(\mathbf{U})$, $J^\sigma_{ijmn}(\mathbf{U})$, and $H^\sigma_{ij}(\mathbf{U})$ are given by

$$I^\sigma_{ijmn}(\mathbf{U})=\left(2k_{iq}\delta_{mp}\delta_{jn}+2k_{jq}\delta_{im}\delta_{pn}-\delta_{im}\delta_{jq}\delta_{pn}-\delta_{iq}\delta_{jn}\delta_{pm}+\delta_{im}\delta_{jn}k_q\frac{\partial}{\partial k_p}\right)\nabla_pU_q,$$

$$E^\sigma_{ijmn}(\mathbf{U})=\left(\delta_{im}\delta_{jq}\delta_{pn}+\delta_{jm}\delta_{iq}\delta_{pn}+\delta_{im}\delta_{jn}k_q\frac{\partial}{\partial k_p}\right)\nabla_pU_q,$$

$$J^\sigma_{ijmn}(\mathbf{U})=\left(2k_{jq}\delta_{im}\delta_{pn}-\delta_{im}\delta_{pn}\delta_{jq}+\delta_{jn}\delta_{pm}\delta_{iq}+\delta_{im}\delta_{jn}k_q\frac{\partial}{\partial k_p}\right)\nabla_pU_q,$$

$$H^\sigma_{ij}(\mathbf{U})=2k_{\rm in}\nabla_jU_n-\nabla_jU_i,$$

where $k_{ij}=k_ik_j/k^2$. The source terms I^f_{ij}, I^h_{ij}, I^g_{ij}, I^F_i, and I^G_i which contain the large-scale spatial derivatives of the mean magnetic field, are given in [11,17] [see also Eqs. (A12) and (A13) below]. Next, in Eqs. (A2)–(A7) we split the tensor for magnetic fluctuations into nonhelical, h_{ij}, and helical,

$h_{ij}^{(H)}$, parts. The helical part of the tensor of magnetic fluctuations $h_{ij}^{(H)}$ depends on the magnetic helicity and it follows from the magnetic helicity conservation arguments (see, e.g., Refs. [18–22,25]). We also use the spectral τ approximation which postulates that the deviations of the third-moment terms, $\hat{\mathcal{N}}M^{(\mathrm{III})}(\mathbf{k})$, from the contributions to these terms afforded by the background turbulent convection, $\hat{\mathcal{N}}M^{(\mathrm{III},0)}(\mathbf{k})$, are expressed through the similar deviations of the second moments, $M^{(\mathrm{II})}(\mathbf{k})-M^{(\mathrm{II},0)}(\mathbf{k})$ [see Eq. (4)].

We take into account that the characteristic time of variation of the mean magnetic field $\mathbf{B}$ is substantially larger than the correlation time $\tau(k)$ for all turbulence scales. This allows us to get a stationary solution for Eqs. (A2)–(A7) for the second-order moments, $M^{(\mathrm{II})}(\mathbf{k})$, which are the sum of contributions caused by a shear-free turbulent convection and a sheared turbulent convection. The contributions to the mean electromotive force caused by a shear-free turbulent convection are given in [17]. On the other hand, the contributions to the mean electromotive force caused by the sheared turbulent convection are $\mathcal{E}_m^{\sigma}=\varepsilon_{mji}\int g_{ij}^{\sigma}(\mathbf{k})d\mathbf{k}$. In particular, in the kinematic approximation the contributions to the cross-helicity tensor g_{ij}^{σ} caused by the sheared turbulent convection, are given by

$$g_{ij}^{\sigma}(\mathbf{k}) = \tau[J_{ijmn}^{\sigma}\tilde{g}_{mn} + I_{ij}^{(g,\sigma)} + g e_n P_{jn}(k) G_i^{\sigma}(-\mathbf{k})], \quad \text{(A8)}$$

where

$$G_i^{\sigma}(\mathbf{k}) = \tau^2(\nabla_m U_i) I_m^G, \quad \text{(A9)}$$

$$\tilde{g}_{ij} = \tau[I_{ij}^g + \tau g e_n P_{jn}(k) I_i^G], \quad \text{(A10)}$$

$$I_{ij}^{(g,\sigma)} = \tau\{[2P_{js}(k)-\delta_{js}]E_{ikmn}^{\sigma}h_{mn}^{(0)} - \delta_{is}I_{kjmn}^{\sigma}f_{mn}^{(0)}\}B_{s,k}, \quad \text{(A11)}$$

$$I_i^G = -\left(\delta_{ij}\delta_{mk} + \frac{1}{2}\delta_{im}k_j\frac{\partial}{\partial k_k}\right)F_m^{(0)}B_{j,k}, \quad \text{(A12)}$$

$$I_{ij}^g = \left([2P_{jn}(k)-\delta_{jn}]h_{ik}^{(0)} - \delta_{in}f_{kj}^{(0)} - \frac{1}{2}k_n\frac{\partial}{\partial k_k}(f_{ij}^{(0)}+h_{ij}^{(0)})\right)B_{n,k}, \quad \text{(A13)}$$

and $B_{i,j}=\nabla_j B_i$. We take into account that in Eq. (A8) the terms with symmetric tensors with respect to the indexes "i" and "j" do not contribute to the mean electromotive force because $\mathcal{E}_m^{\sigma}=\varepsilon_{mji}\int g_{ij}^{\sigma}(\mathbf{k})d\mathbf{k}$. For the integration in $\mathbf{k}$-space we must specify a model for the background shear-free turbulent convection (with $\mathbf{B}=0$), which is determined by Eqs. (5)–(7) in Sec. III.

The contributions to the mean electromotive force caused by the sheared turbulent convection, are $\mathcal{E}_i^{\sigma}=b_{ijk}^{\sigma}\nabla_k B_j$, where the tensor $b_{ijk}^{\sigma}=b_{ijk}^{u}+b_{ijk}^{F}$ is given by Eqs. (8) and (9) in Sec. III. For derivation of Eqs. (8) and (9) we use the following identities:

$$\int \frac{k_i k_j k_m k_n}{k^4}\sin\theta d\theta d\varphi = \frac{4\pi}{15}\Delta_{ijmn},$$

$$\int \frac{k_i k_j k_m k_n k_p k_q}{k^6}\sin\theta d\theta d\varphi = \frac{4\pi}{105}\Delta_{ijmnpq},$$

and

$$\Delta_{ijmn} = \delta_{ij}\delta_{mn} + \delta_{im}\delta_{nj} + \delta_{\mathrm{in}}\delta_{mj},$$

$$\Delta_{ijmnpq} = \Delta_{mnpq}\delta_{ij} + \Delta_{jmnq}\delta_{ip} + \Delta_{imnq}\delta_{jp} + \Delta_{jmnp}\delta_{iq} + \Delta_{imnp}\delta_{jq} + \Delta_{ijmn}\delta_{pq} - \Delta_{ijpq}\delta_{mn},$$

and

$$\varepsilon_{ikm}e_{ns}\Delta_{jpqmns}\nabla_p U_q = 2\varepsilon_{ikm}[(\partial U)_{mj} + 2e_{mp}\nabla_p U_j + 2e_{qj}\nabla_m U_q],$$

$$\varepsilon_{inm}e_{ms}\Delta_{jkpqns}\nabla_p U_q = 4\{\varepsilon_{inm}[e_{mk}(\partial U)_{nj} + e_{mj}(\partial U)_{kn}] + e_{mn}[\varepsilon_{ikm}(\partial U)_{nj} + \varepsilon_{ijm}(\partial U)_{kn}]\},$$

$$\varepsilon_{inq}e_{ms}\Delta_{jkpmns}\nabla_p U_q = \varepsilon_{inq}[\nabla_p U_q(2e_{mk}\Delta_{jpmn} + 2e_{mj}\Delta_{kpmn} + \Delta_{jpkn}) - \nabla_n U_q(\delta_{jk} + 2e_{jk})],$$

$$\varepsilon_{ijn}e_{ms}\Delta_{kpqmns}\nabla_p U_q = 2\{\varepsilon_{ijn}[2e_{mk}(\partial U)_{nm} + 2e_{mn}(\partial U)_{km} + (\partial U)_{kn}] + \varepsilon_{ijk}e_{mn}(\partial U)_{nm}\}.$$

In Eqs. (8) and (9) we have taken into account only the terms which contribute to the shear-current effect. In particular, we consider the mean magnetic field in a most simple form $\mathbf{B}=(B_x(z),B_y(z),0)$ and we take into account that $B_y \gg B_x$ and $S\tau_0 \ll 1$, where the mean velocity shear is $\mathbf{U}=(0,Sx,0)$ and $\mathbf{W}=(0,0,S)$. Straightforward calculations using Eqs. (8) and (9) and Eqs. (13) and (14) yield $\sigma_B=\sigma_B^u+\sigma_B^F$, where

$$\sigma_B^u = \frac{(q-1+\mu)^2}{15(q-1+2\mu)(q-1)}[1-\mu+\epsilon(1+2\mu)], \quad \text{(A14)}$$

$$\sigma_B^F = \frac{a_*(q-1+\mu)^3}{35(q-1+3\mu)(q-1)^2}[2+3(2-3\mu)\sin^2\phi], \quad \text{(A15)}$$

ϕ is the angle between the unit vector $\mathbf{e}$ and the background vorticity $\mathbf{W}$ due to the large-scale shear. Equations (A14) and (A15) yield Eq. (15) given in Sec. III.

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